4725 Further Pure Mathematics 1

1	 $\frac{7}{26} + \frac{17}{26}\text{i}$.	M1 A1 A1 A1	 4 **4**	Multiply by conjugate of denominator Obtain correct numerator Obtain correct denominator
2	**(i)** $\frac{1}{10}\begin{pmatrix} 5 & 0 \\ -a & 2 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 3 & -2 \\ 2a & 6 \end{pmatrix}$	B1 B1 B1 B1	 2 2 **4**	Both diagonals correct Divide by correct determinant Two elements correct Remaining elements correct
3	$n^2(n+1)^2 + n(n+1)(2n+1) + n(n+1)$ $n(n+1)^2(n+2)$	M1 A1 A1 M1 A1ft A1	 6 **6**	Express as sum of 3 terms 2 correct unsimplified terms 3rd correct unsimplified term Attempt to factorise Two factors found, ft their quartic Correct final answer a.e.f.
4	 $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	B1 M1 A1 A1	 4 **4**	State or use correct result Combine matrix and its inverse Obtain **I** or $\mathbf{I}^2$ but not 1 Obtain zero **matrix** but not 0 **S.C. If 0/4, B1 for $\mathbf{AA}^{-1} = \mathbf{I}$**
5	*Either* $4k - 4$ $k = 1$ *Or*	M1 M1 A1 M1 A1ft M1 A1 M1 A1 A1	 5 **5**	Consider determinant of coefficients of LHS Sensible attempt at evaluating any 3×3 det Obtain correct answer a.e.f. unsimplified Equate det to 0 Obtain $k = 1$, ft provided all M's awarded Eliminate either x or y Obtain correct equation Eliminate 2nd variable Obtain correct linear equation Deduce that $k = 1$
6	**(i)** *Either* *Or* **(ii)** **(iii)** $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ **(iv)**	B1 DB1 B1 DB1 B1 DB1 B1 B1 B1B1B1	2 2 2 3 **9**	Reflection, in x-axis Stretch parallel to y-axis, s.f. –1 Reflection, in $y = -x$ Each column correct Rotation, 90°, clockwise about O **S.C. If (iii) incorrect, B1 for identifying their transformation, B1 all details correct**

7	**(i)** $13^n + 6^{n-1} + 13^{n+1} + 6^n$ **(ii)**	B1 M1 A1 B1 B1 B1 B1	 3 4 **7**	Correct expression seen Attempt to factorise both terms in (i) Obtain correct expression Check that result is true for n =1 (or 2) Recognise that (i) is divisible by 7 Deduce that u_{n+1} is divisible by 7 Clear statement of Induction conclusion
8	**(i)** **(ii)** $\alpha + \beta = 6k, \alpha\beta = k^2$ $\alpha - \beta = (4\sqrt{2})k$ **(iii)** $\sum \alpha' = 6k$ $\alpha' \beta' = \alpha\beta - (\alpha - \beta) - 1$ $\alpha' \beta' = k^2 - (4\sqrt{2})k - 1$ $x^2 - 6kx + k^2 - (4\sqrt{2})k - 1 = 0$	M1 A1 B1 B1 M1 A1 B1ft M1 A1ft B1ft	 2 4 4 **10**	Expand at least 1 of the brackets Derive given answer correctly State or use correct values Find value of $\alpha - \beta$ using (i) Obtain given value correctly (allow if $-6k$ used) Sum of new roots stated or used Express new product in terms of old roots Obtain correct value for new product Write down correct quadratic **equation**
9	**(i)** **(ii)** $1 + \frac{1}{3} - \frac{1}{2n-1} - \frac{1}{2n+1}$ **(iii)** $\frac{4}{3}$	M1 A1 M1 M1 A1 A1 M1 A1 B1ft	 2 6 1 **9**	Use correct denominator Obtain given answer correctly Express terms as differences using (i) Do this for at least 1st 3 terms First 3 terms all correct Last 3 terms all correct (in terms or n or r) Show pairs cancelling Obtain correct answer, a.e.f.(in terms of n) Given answer deduced correctly, ft their (ii)

10	**(i)** $x^2 - y^2 = 2, 2xy = \sqrt{5}$	M1 A1		Attempt to equate real and imaginary parts Obtain both results a.e.f.
	$4x^4 - 8x^2 - 5 = 0$	M1 M1		Eliminate to obtain quadratic in x^2 or y^2 Solve to obtain x (or y) values
	$x = \pm\frac{\sqrt{10}}{2}, y = \pm\frac{\sqrt{2}}{2}$	A1		Correct values for both x & y obtained a.e.f.
	$\pm(\frac{\sqrt{10}}{2} + \mathrm{i}\frac{\sqrt{2}}{2})$	A1	6	Correct answers as complex numbers
	(ii) $z^2 = 2 \pm \mathrm{i}\sqrt{5}$	M1 A1		Solve quadratic in z^2 Obtain correct answers
	$z = \pm(\frac{\sqrt{10}}{2} \pm \mathrm{i}\frac{\sqrt{2}}{2})$	M1 A1ft	4	Use results of (i) Obtain correct answers, ft must include root from conjugate
	(iii)	B1ft	1	Sketch showing roots correctly
	(iv)	B1 B1ft B1ft	3 **14**	Sketch of straight line, $\perp$ to α Bisector